# Report on *"Food distribution by a food bank among local social solidarity institutions"*

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Executive Summary: A food bank is a non-profit, social solidarity organization that typically distributes the donated food among a wide variety of local non-profit, social solidarity institutions that in turn feed the low-income people. The problem presented by the Portuguese Federation of Food Banks is to determine, for a specific food bank, the quantities of the donated products that have to be assigned to each local social solidarity institution in order to satisfy the needs of the supported people as much as possible, without favouring any institution. We propose a linear programming model followed by a rounding heuristic to obtain a solution to the problem. Computational results have not yet been reported.

## 1 Introduction

Food banks are non-profit, social solidarity organizations that typically distribute the donated food among local non-profit, social solidarity institutions that in turn distribute the food among the people in need. The first food bank in Portugal started in 1992 in Lisbon and ever since many food banks have been opened. Today, the Portuguese Federation of Food Banks represents a network of 17 food banks across the country. Each food bank runs a centralized warehouse which serves as a single collection and distribution point for food donations. The largest portion of the donated food comes from food leftover from the normal processes of profit-making companies. This food can come from any part of the food chain, e.g. from growers who have produced too much or whose food is not that visually appealing, from manufacturers who have overproduced, or from retailers who have over-ordered. All these donated products are delivered to the food bank's warehouses by the companies themselves. Often the products are approaching or past their expiry dates and, in these cases, the food banks should ensure that the food is safe to be consumed. The other portion of the donated products comes from the general public through the national seasonal campaigns where each food bank collects food at various local supermarkets. A social solidarity institution has to be registered in a food bank to receive food from it. The institutions must go to the food banks to collect the products.

The problem presented by the Portuguese Federation of Food Banks is to determine, for a particular food bank, the quantities of the donated products that have to be assigned to each local social solidarity institution in order to satisfy the needs of the supported people as much as possible, without privileging any one institution. There follow details of the problem and then we describe how the Lisbon's food bank, the largest of its kind in Portugal, solves it in practice. We should add that this procedure is also followed by the majority of the other national food banks.

The supply of the donated products to the institutions is planned according to the types of the products involved *i.e.* dry or fresh. A *dry product* is considered to be every product that is non-perishable in the short term (e.g., rice, pasta, canned food and UHT milk), whereas a *fresh product* is the opposite (e.g., yogurt, cheese, butter, pasteurized milk, fruit, fresh vegetables and frozen food). Each institution receives one package (the so-called box) of dry products once a month and one box of fresh products once a week at the most. There is a schedule (date and time) to pick up the dry and the fresh products. The day for the dry products is also used to collect fresh products. An institution might not manage to go to the food bank every week for the fresh products (e.g., there is no vehicle or staff). Some food banks do not collect food that requires refrigeration or freezing, such as yogurt, butter, cheese and frozen products, because their warehouses have no refrigerator systems.

By the 20th of every month, each institution sends a form with the following information relative to the next month:

- number of individuals that receive packages (the so-called baskets) with dry products (I);
- number of individuals that receive baskets with fresh products (II);
- numbers of days the institution serves breakfasts (III), lunches (IV), snacks (V) and dinners (VI);
- numbers of individuals that take breakfasts (VII), lunches (VIII), snacks (IX) and dinners (X);

- list of products not wanted (XI);
- numbers of families and children that receive baskets with dry products;
- numbers of families and children that receive baskets with fresh products;
- numbers of children, elderly, other individuals and employees that take breakfasts, lunches, snacks and dinners;
- daily number of customers;
- specific observations.

Data (I) to (XI) are used to find a solution to the food distribution problem, while the remaining data are only used for statistical purposes.

To plan food distribution among the institutions, the food bank classifies the products into two categories: products for breakfast and snacks and products for lunch and dinner. A *product for breakfast and snacks* is considered to be every product that is typically consumed at breakfast-time and snacks (e.g., milk, cookies and sugar), and a *product for lunch and dinner* is considered to be every product that is typically consumed at lunches and dinner-times (e.g., meat, canned fish, pasta and rice). An institution either receives products from both categories (an Agreement A institution) or just receives products for breakfast and snacks (an Agreement B institution). In general, an institution of Agreement B category serves only breakfasts and snacks or has inadequate credibility for the food bank to receive all kinds of products. There are also institutions that receive only a surplus of the fresh products (Agreement C institutions), since, as a rule, the food bank does not have enough food to meet the needs of the other institutions. Agreement C institutions wait for one of the other agreements.

At the end of each month, the food bank plans the supply of dry products in stock for the next month among all institutions with Agreement A or B. The supply of the fresh products in stock is planned on a daily basis for all institutions that are scheduled for the day. We then describe the procedure followed by the food bank to plan the dry food distribution. Distribution of the fresh food is planned in a similar way, except that one part of the fresh products in stock is reserved for the Agreement C institutions.

**Procedure to plan the dry food distribution** for a specific month

For each Agreement A institution $j$, define the number of individuals that receive baskets with dry products according to the risk of hunger of the supported people, $C_j^1$, where $C_j^1 = (I) \times FC$ with $FC = 1.15$ for a higher risk (the so-called Agreement AI), $FC = 0.85$ for a lower risk (Agreement AIII) or $FC = 1$ for an intermedium level (Agreement AII). Also define the number of individuals that receive baskets with products for breakfast and snacks, $C_j^{1,PBS}$, and the number of individuals that receive baskets with products for lunch and dinner, $C_j^{1,PLD}$, as $C_j^{1,PBS} = C_j^{1,PLD} = C_j^1$.

For each institution $j$, calculate:

- the mean daily number of "meals that use products for breakfast and snacks only", $C_j^{2,PBS}$,

$$C_j^{2,PBS} = \frac{0.3 \times (VII) \times (III) + 0.2 \times (VIII) \times (IV) + 0.3 \times (IX) \times (V) + 0.2 \times (X) \times (VI)}{30};$$

- the mean daily number of "meals that use products for lunch and dinner only", $C_j^{2,PLD}$,

$$C_j^{2,PLD} = \frac{0.5 \times (VIII) \times (IV) + 0.5 \times (X) \times (VI)}{30};$$

- the sums $C_j^{PBS} = C_j^{1,PBS} + C_j^{2,PBS}$ and $C_j^{PLD} = C_j^{1,PLD} + C_j^{2,PLD}$.

Calculate the sums $C^{PBS} = \sum_j C_j^{PBS}$ and $C^{PLD} = \sum_j C_j^{PLD}$.

For each institution $j$, calculate the quantity of product $p$ to supply:

- if $p$ is a product for breakfast and snacks, $\frac{C_j^{PBS}}{C^{PBS}} \times d_p$,
- if $p$ is a product for lunch and dinner, $\frac{C_j^{PLD}}{C^{PLD}} \times d_p$,

where $d_p$ is the quantity of product $p$ in stock (by the end of the previous month), measured in kg. ■

The food bank makes some slight adjustments to the plan resulting from this procedure (e.g., the products not wanted by the institutions (XI) are not supplied or the institutions that support more children and elderly people can receive larger quantities of baby food than the others).

The main drawback to that procedure is that the dietary needs of the individuals in need are not explicitly catered for. To solve the food distribution problem presented by the Portuguese Federation of Food Banks, we propose a linear programming approach where this issue is taken into consideration. The optimal linear solution obtained is then rounded to integer by a heuristic. In reviewing the literature we found no work in optimal food distribution by food banks. There exist some technical reports on food crisis or food distribution (see for example [4, 7]), but we have not found any recent or older scientific research work that uses mathematical tools such as optimization techniques (e.g., linear and integer programming). For general discussions on linear and integer programming, we refer those interested to [1, 3].

## 2 Formulation

In this section we present a linear programming approach for the food distribution problem described above. This problem is solved in two stages, where the dietary needs of the supported people are considered. The distribution of the dry products is modeled in the first stage. The second stage concerns distribution of the fresh products bearing in mind the needs already met by the previous distributions over the month. In both stages, it is assumed that each individual receives either meals or baskets, and an individual that receives a basket containing dry products also receives a basket with fresh food and vice-versa. We consider three types of individuals each one with specific dietary needs: the *child*, the *elderly* and *others*. Account is taken of the needs for the main macro nutrients and energy for each type of individual per type of meal (*breakfast*, *lunch*, *snack* and *dinner*). We also bear in mind the needs for the main micro nutrients for each type of individual a day. For the micro nutrients, it is meaningless to consider the needs per type of meal.

**General data**

$$IT = \{\text{institutions that receive products}\}$$
$$P = \{\text{donated products}\}$$
$$NM = \{\text{main macro nutrients and energy}\}$$
$$Nm = \{\text{main micro nutrients}\}$$
$$I = \{\text{types of individuals}\}$$
$$M = \{\text{types of meals}\}$$

$a_{np}$ – quantity of nutrient $n \in NM \cup Nm$ per unit of product $p \in P$

$qm_{nmi}$ – mean quantity of nutrient $n \in NM$ that a meal of type $m \in M$ should contain for an individual of type $i \in I$

$q'_{ni}$ – mean quantity of nutrient $n \in Nm$ that an individual of type $i \in I$ should consume a day

$q_{ni}$ – mean quantity of nutrient $n \in NM \cup Nm$ that an individual of type $i \in I$ should consume a month

$\delta_j$ – mean proportion of each nutrient that every basket delivered by institution $j \in IT$ should contain, depending on the risk of hunger of the supported people. For the institutions which do not deliver baskets, this parameter is null.

The products are grouped according to their similarities in terms of composition in nutrients (e.g., oil and olive oil define one group),

$$GP_g = \{p \in P : \ p \in \text{group } g\}, \ g = 1, \cdots, ngd, \ \text{where } ngd \text{ is the number of groups.}$$

Considering specific dietary needs, we define the set of special products (e.g., baby food is special),

$$SP = \{p \in P : \ p \text{ is special for some type of individual } i \in I\}$$

and for each product $p \in SP$ there is a subset of types of individuals for whom $p$ is special (e.g., baby food can be special for children and the elderly),

$$SI_p = \{i \in I : \ p \text{ is a special product for individuals of type } i\}.$$

## 2.1 Dry product distribution

Every institution has a specific day each month to go to the food bank to pick up the dry products. It is assumed that a dry product can be donated to an institution provided its expiry date has not yet lapsed by $l$ days after the pick-up day. The dry product distribution for each month is planned by considering the dietary needs of the supported people from each institution for a period of one month.

**Data** for a specific month

$DP = \{$dry products available$\}$

$ID = \{$institutions that receive dry products$\}$

$DU_j = \{$dry products not wanted by institution $j\}$, $j \in ID$

$d_p$ – quantity available of product $p \in DP$ (measured in kg or l)

$val_p$ – expiry date of product $p \in DP$

$datD_j$ – date when institution $j \in ID$ picks up the dry products

$ndm_j$ – number of days when institution $j \in ID$ offers meals between $datD_j$ (exclusively) and the next pick-up day of dry products (inclusively)

$nm_{mij}$ – number of meals of type $m \in M$ served each day to individuals of type $i \in I$ by institution $j \in IT$

$nb_{ij}$ – number of individuals of type $i \in I$ that receive baskets from institution $j \in IT$

Other data: the set of institutions that support individuals of type $i \in SI_p$ (*i.e.* with special needs regarding product $p$),

$$SIT_p = \left\{ j \in IT : \sum_{i \in SI_p} \left( nb_{ij} + \sum_{m \in M} nm_{mij} \right) > 0 \right\}.$$

**Variables**

$xd_{pj}$ – quantity of product $p \in DP$ received by institution $j \in ID$ (kg or l)

$wd_n$ – proportion of nutrient $n \in NM \cup Nm$ received by every institution in $ID$

$zd_p$ – relative quantity of product $p \in SP \cap DP$ received by institution $j \in SIT_p \cap ID$ (kg or l)

**Formulation**

$$\max \sum_{n \in NM \cup Nm} \sum_{p \in DP} \sum_{\substack{j \in ID: \\ p \in DP \setminus DU_j}} a_{np} xd_{pj} \tag{1}$$

subject to

$$\sum_{\substack{p \in DP \setminus DU_j: \\ val_p \geq datD_j + l}} a_{np} xd_{pj} \leq \sum_{i \in I} \left( \delta_j nb_{ij} q_{ni} + \sum_{m \in M} nm_{mij} qm_{nmi} ndm_j \right), \forall n \in NM, \forall j \in ID \tag{2}$$

$$\sum_{\substack{p \in DP \setminus DU_j: \\ val_p \geq datD_j + l}} a_{np} xd_{pj} \leq \sum_{i \in I} \left( \delta_j nb_{ij} q_{ni} + \frac{q'_{ni}}{4} \left( \sum_{m \in M} nm_{mij} \right) ndm_j \right), \forall n \in Nm, \forall j \in ID \tag{3}$$

$$\sum_{\substack{p \in DP \setminus DU_j: \\ val_p \geq datD_j + l}} a_{np} xd_{pj} = wd_n \sum_{i \in I} \left( \delta_j nb_{ij} q_{ni} + \sum_{m \in M} nm_{mij} qm_{nmi} ndm_j \right), \forall n \in NM, \forall j \in ID \tag{4}$$

$$\sum_{\substack{p \in DP \setminus DU_j: \\ val_p \geq datD_j + l}} a_{np} xd_{pj} = wd_n \sum_{i \in I} \left( \delta_j nb_{ij} q_{ni} + \frac{q'_{ni}}{4} \left( \sum_{m \in M} nm_{mij} \right) ndm_j \right), \forall n \in Nm, \forall j \in ID \tag{5}$$

$$xd_{pj} \leq \frac{d_p}{\sum_{p \in GP_g \cap DP} d_p} \sum_{p \in GP_g \cap DP} xd_{pj}, \; g = 1, ..., ngd, \forall p \in GP_g \cap DP, \forall j \in ID \tag{6}$$

$$xd_{pj} = zd_p \sum_{i \in SI_p} \left( nb_{ij} + \sum_{m \in M} nm_{mij} ndm_j \right), \forall p \in SP \cap DP, \forall j \in SIT_p \cap ID \tag{7}$$

$$\frac{\sum_{j \in ID \setminus SIT_p} xd_{pj}}{\sum_{j \in ID \setminus SIT_p} \sum_{i \in I} \left( nm_{ij} + \sum_{m \in M} nm_{mij} ndm_j \right)} \leq \frac{\sum_{j \in SIT_p \cap ID} xd_{pj}}{\sum_{j \in SIT_p \cap ID} \sum_{i \in SI_p} \left( nb_{ij} + \sum_{m \in M} nm_{mij} ndm_j \right)}, \quad \forall p \in SP \cap DP \tag{8}$$

$$\sum_{\substack{j \in ID: p \in DP \setminus DU_j \\ val_p \geq datD_j + l}} xd_{pj} \leq d_p, \forall p \in DP \tag{9}$$

$$xd_{pj} \geq 0, \forall p \in DP, \forall j \in ID : (p \in DP \setminus DU_j) \wedge (val_p \geq datD_j + l) \tag{10}$$

$$wd_n \geq 0, \forall n \in NM \cup Nm \tag{11}$$

$$zd_p \geq 0, \forall p \in DP. \tag{12}$$

Expression (1) states the objective of maximizing the total quantity of nutrients received by the institutions over the month. The right-hand sides of constraints (2) and (3) define the amount of each nutrient needed by each institution, according to the number of individuals of each type that receive baskets or meals and the number of days with meals. With these constraints, it is intended that the quantity of each nutrient received by each institution does not exceed that amount. Constraints (2) refer to the macro nutrients and energy and constraints (3) to the micro nutrients. Since, in general, the amount of donated products is insufficient to supply all the dietary needs, we aim to construct a set of constraints that conveys the sense of justice and equality that the food bank practises. Constraints (4) and (5) ensure that the proportion of each nutrient received (macro nutrient/energy and micro nutrient, respectively) is the same for all institutions. In constraints (6), we consider the groups of products that are nutritionally similar. For every group, it is desirable that the proportion of the amount of each product received by each institution (considering the total amount of all products from the group received by the institution) does not exceed the similar proportion regarding the food bank. With these constraints we try to avoid solutions where institutions receive only one product from the same group. In constraints (7) and (8) we consider the products that are special for certain types of individuals. According to constraints (7), the ratio of the amount of each special product received by each institution over the number of baskets and meals for individuals with special needs must be the same for all institutions with the same type of needs. By imposing constraints (8), we try to obtain a distribution where institutions supporting individuals with special needs globally receive a larger quantity of each special product than the other institutions. Hence, these constraints require that the ratio of the total amount of each special product received by the institutions with no special needs over the total number of baskets and meals does not exceed the similar ratio regarding the other institutions. An obvious restriction is that the amount of a product received by all institutions does not exceed the available amount of the product (constraints (9)). The remaining restrictions state non-negativity requirements on variables.

As an alternative objective function, we propose to maximize the sum of the proportions of the nutrients received by the institutions over the month. Then, instead of (1), we will have

$$\max \sum_{n \in NM \cup Nm} wd_n. \tag{13}$$

Since we consider the sum of proportions, the distribution obtained may be such that some institutions receive large proportions of certain nutrients and small proportions of others. The same can happen with the objective function (1): we may obtain a solution where there is a shortage of certain nutrients delivered to the institutions. Nevertheless, if that happens all institutions are affected because of the equilibrium constraints (4) and (5).

## 2.2 Fresh product distribution

Every institution has a specific day each week to go to the food bank to pick up the fresh food. It is assumed that a fresh product can be donated to an institution provided its expiry date is greater than or equal to $h$ days after the pick-up day. The formulation is similar to the one described for the dry product distribution.

**Data** for a specific day $d$

$$FP = \{\text{fresh products available at the beginnig of } d\}$$

$$IF = \{\text{institutions that receive fresh products during } d\}$$

$$UF_j = \{\text{fresh products not wanted by institution } j\},\ j \in IF$$

$$d_p - \text{quantity available of product } p \in FP \text{ (kg or l)}$$

$$val_p - \text{expiry date of product } p \in FP$$

**Variables**

$xf_{pj}$ – quantity of product $p \in FP$ received by institution $j \in IF$ (kg or l)

$wf_n^d$ – proportion of nutrient $n \in NM \cup Nm$ received by every institution in $IF$

$zf_p$ – relative quantity of product $p \in SP \cap FP$ received by institution $j \in SIT_p \cap IF$ (kg or l)

**Formulation**

$$\max \sum_{n \in NM \cup Nm} \sum_{p \in FP} \sum_{\substack{j \in IF: \\ p \in FP \setminus FU_j}} a_{np} xf_{pj} \tag{14}$$

subject to

$$\sum_{\substack{p \in FP \setminus FU_j: \\ val_p \geq d+h}} a_{np} xf_{pj} \leq \sum_{i \in I} \left( \delta_j nb_{ij} q_{ni} + \sum_{m \in M} nm_{mij} qm_{nmi} ndm_j \right) (1 - pp_n), \quad \forall n \in NM, \forall j \in IF \tag{15}$$

$$\sum_{\substack{p \in FP \setminus FU_j: \\ val_p \geq d+h}} a_{np} xf_{pj} \leq \sum_{i \in I} \left( \delta_j nb_{ij} q_{ni} + \frac{q'_{ni}}{4} \left( \sum_{m \in M} nm_{mij} \right) ndm_j \right) (1 - pp_n), \quad \forall n \in Nm, \forall j \in IF \tag{16}$$

$$\sum_{\substack{p \in FP \setminus FU_j: \\ val_p \geq d+h}} a_{np} xf_{pj} = wf_n^d \sum_{i \in I} \left( \delta_j nb_{ij} q_{ni} + \sum_{m \in M} nm_{mij} qm_{nmi} ndm_j \right) (1 - pp_n), \quad \forall n \in NM, \forall j \in IF \tag{17}$$

$$\sum_{\substack{p \in FP \setminus FU_j: \\ val_p \geq d+h}} a_{np} xf_{pj} = wf_n^d \sum_{i \in I} \left( \delta_j nb_{ij} q_{ni} + \frac{q'_{ni}}{4} \left( \sum_{m \in M} nm_{mij} \right) ndm_j \right) (1 - pp_n), \quad \forall n \in Nm, \forall j \in IF \tag{18}$$

$$xf_{pj} \leq \frac{d_p}{\sum\limits_{p \in GP_g \cap FP} d_p} \sum_{p \in GP_g \cap FP} xf_{pj},\ g = 1, ..., ngd,\ \forall p \in GP_g \cap FP,\ \forall j \in IF \tag{19}$$

$$xf_{pj} = zf_p \sum_{i \in SI_p} \left( nb_{ij} + \sum_{m \in M} nm_{mij} ndm_j \right),\ \forall p \in SP \cap FP,\ \forall j \in SIT_p \cap IF \tag{20}$$

$$\frac{\sum\limits_{j \in IF \setminus SIT_p} xf_{pj}}{\sum\limits_{j \in IF \setminus SIT_p} \sum\limits_{i \in I} \left( nm_{ij} + \sum\limits_{m \in M} nm_{mij} ndm_j \right)} \leq \frac{\sum\limits_{j \in SIT_p \cap IF} xf_{pj}}{\sum\limits_{j \in SIT_p \cap IF} \sum\limits_{i \in SI_p} \left( nb_{ij} + \sum\limits_{m \in M} nm_{mij} ndm_j \right)},\quad \forall p \in SP \cap FP \tag{21}$$

$$\sum_{\substack{j \in IF : p \in FP \setminus FU_j \\ val_p \geq d+h}} xf_{pj} \leq d_p,\ \forall p \in FP \tag{22}$$

$$xf_{pj} \geq 0,\ \forall p \in FP,\ \forall j \in IF : (p \in FP \setminus FU_j) \wedge (val_p \geq d + h) \tag{23}$$

$$wf_n^d \geq 0,\ \forall n \in NM \cup Nm \tag{24}$$

$$zf_p \geq 0,\ \forall p \in FP, \tag{25}$$

where $pp_n$ is the proportion of nutrient $n$ already received by every institution. This value can be obtained by totaling the proportions that were delivered during the month through the dry products ($wd_n$) and the fresh ones ($wf_n^{d'}$, $\forall d' < d$).

The right-hand sides of constraints (15) and (16) define the amount of each nutrient still needed by each institution, according to the number of individuals of each type that receive baskets or meals, the number of days with meals and the needs already met by the previous distributions of dry and fresh products. In constraints (17) and (18), the proportion of each nutrient received by each institution is relative to the amount still needed. Expression (14) and constraints (15) to (25) have the same meaning for the fresh products as the objective function (1) and constraints (2) to (12) for the dry products respectively.

# 3 Rounding heuristics

In general, the donated products are packed. But, the decision variables described above represent quantities measured in metric units of weight (Kg) or capacity (l). Therefore, in the optimal solutions to the formulations, the quantity of a product received by an institution will probably correspond to a fractional number of packages. We propose two alternative simple heuristics to round these fractional values to integer. Observe that we could formulate the problems in integer programming where the decision variables would represent integer numbers of packages. Nevertheless, in this case, the equality constraints (4), (5) and (7) (dry product distribution) or (17), (18) and (20) (fresh product distribution) should be replaced by less strict constraints, otherwise the problems would be infeasible. In addition, such integer programming models may be far more difficult to solve than each linear programming formulation together with a rounding heuristic.

For the packed products, it is therefore necessary to translate the optimal quantities into numbers of packages. Let $y_{pj}$ be the optimal number of packages of product $p$ for institution $j$. First of all, $\lfloor y_{pj} \rfloor$ packages of product $p$ are assigned to institution $j$. Then, a rounding heuristic is applied to decide if one more package of product $p$ is delivered or not to institution $j$, or equivalently, if the (non-vanished) rest $y_{pj} - \lfloor y_{pj} \rfloor$ is rounded to one or zero, respectively. We describe the heuristics for the dry products, but those for the fresh ones are similar. Let $\varepsilon_{pj} = y_{pj} - \lfloor y_{pj} \rfloor$ be the fraction of package with product $p$ that has not yet been assigned to institution $j$, and $R_p = \sum_{j \in ID} \varepsilon_{pj}$ be the sum of these fractions over all institutions. Note that if a packed product $p$ is totally assigned to the institutions, then $R_p$ is integer. If $R_p$ was not integer then $\sum_{j \in ID} y_{pj}$, the total number of packages of product $p$, would be fractional. In simpler terms, consider that all products are packed.

*Heuristic A* performs roundings by merely considering the fractions of packages that have not yet been assigned to the institutions. This heuristic can be summarized as follows: for each (packed) product $p$, select an institution by the decreasing order of $\varepsilon_{pj}$, assign one more package to this institution and repeat the whole process among the remaining institutions until there is no product.

**Heuristic A**

**Input** $\varepsilon_{pj}$ and $y_{pj}$, $\forall p \in DP, j \in ID$ and $R_p$, $\forall p \in DP$

$\Omega_p \longleftarrow \{(p, j) : j \in ID,\ \varepsilon_{pj} > 0\}$, $\forall p \in DP$;

**for** $p \in DP$ **do**

  **if** $\Omega_p \neq \emptyset$ **then**

    sort $\Omega_p$ by decreasing order of $\varepsilon_{pj}$;

    **while** $\Omega_p \neq \emptyset$ **do**

      select the first pair of $\Omega_p$, $(p, j_1)$;

      $y_{pj_1} \longleftarrow \lfloor y_{pj_1} \rfloor + 1$;

      $R_p \longleftarrow R_p - 1$;

      **if** $R_p = 0$ **then**

        $\Omega_p \longleftarrow \emptyset$;

      **else** $\Omega_p \longleftarrow \Omega_p \setminus \{(p, j_1)\}$;

Let $\text{needs}_{nj}$ be the amount of nutrient $n$ needed by institution $j$ (right-hand sides of constraints (2) and (3)) and $\delta_{nj} = \dfrac{\sum_{p \in DP} a_{np}\varepsilon_{pj}}{\text{needs}_{nj}}$ be the relative needs of institution $j$ for nutrient $n$ that have not yet been satisfied. *Heuristic B* performs roundings taking into account these needs. The heuristic can be summarized as follows: select a nutrient and an institution by the decreasing order of $\delta_{nj}$, assign to this institution one more package of the product richest in that nutrient, actualize all $\delta_{nj}$ for the institution and repeat the whole process until there are no products. Let $\wp_n = \{p \in DP : a_{np} > 0 \text{ and } R_p > 0\}$ be the set of all non assigned products containing nutrient $n$, sorted by decreasing order of $a_{np}$. In simpler terms, suppose that the capacities of all packages of each product are the same. Let $a'_{np}$ be the quantity of nutrient $n$ per package of product $p$.

**Heuristic B**

**Input** $\delta_{nj}, \forall n \in NM \cup Nm, j \in ID$; $y_{pj}, \forall p \in DP, j \in ID$; $a'_{np}, \forall n \in NM \cup Nm, p \in DP$; $\wp_n, \forall n \in NM \cup Nm$ and $R_p, \forall p \in DP$

$\Phi \longleftarrow \{(n,j) : n \in NM \cup Nm,\ j \in ID,\ \delta_{nj} > 0\}$;

**while** $\Phi \neq \emptyset$ **do**

sort $\Phi$ by decreasing order of $\delta_{nj}$;

select the first pair of $\Phi$, $(n_1, j_1)$;

select the first element of $\wp_{n_1}$, $p_1$;

$y_{p_1 j_1} \longleftarrow \lfloor y_{p_1 j_1} \rfloor + 1$;

$R_{p_1} \longleftarrow R_{p_1} - 1$;

$\delta_{nj_1} \longleftarrow \delta_{nj_1} - \frac{a'_{np_1}}{\text{needs}_{nj_1}}, \forall n \in NM \cup Nm : a_{np_1} > 0$;

$\Phi \longleftarrow \Phi \setminus \{(n, j_1),\ n \in NM \cup Nm,\ \delta_{nj_1} \leq 0\}$;

**if** $R_{p_1} = 0$ **then**

$\wp_n \longleftarrow \wp_n - p_1, \forall n \in NM \cup Nm : a_{np_1} > 0$;

# 4 Results

The Lisbon's food bank has provided the plan that was defined at the end of February 2010 for the distribution of its dry products over the following month. The number of registered Agreement A or B institutions was 340 and the number of dry products in stock about 28. We had no information about the expiry dates of the products. We therefore assumed that every product could be delivered throughout the month. Data for the fresh product distribution were not provided. We considered three macro nutrients, carbohydrate, protein and fat, and five micro nutrients, calcium, iron, magnesium, phosphorus and vitamin B12. Information about food composition was taken from [5] and the calculations of the needs for nutrients per type of individual (Tables 1 and 2) were based on data from [2]. We considered an individual up to the age of eight (inclusively) to be a child and a person above seventy years to be an elderly one. This classification might not coincide with those used by the institutions. The number of variables and constraints of the formulation for the dry product distribution in March 2010 is about 10000 and 12000, respectively.

| | Child | | | | Elderly | | | | Others | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nutrient | B. | L. | S. | D. | B. | L. | S. | D. | B. | L. | S. | D. |
| Carbohydrate (g) | 46.8 | 81.8 | 46.8 | 58.5 | 49.5 | 19.4 | 86.6 | 61.9 | 57.8 | 101.1 | 57.8 | 72.1 |
| Protein (g) | 14.9 | 26 | 14.9 | 18.6 | 15.8 | 27.6 | 15.8 | 19.6 | 18.4 | 32.2 | 18.4 | 22.9 |
| Fat (g) | 10.4 | 18.2 | 10.4 | 13 | 11 | 19.4 | 11 | 13.7 | 12.8 | 22.4 | 12.8 | 16.1 |
| Energy (Kcal) | 340 | 595 | 340 | 425 | 360 | 630 | 360 | 450 | 420 | 735 | 420 | 525 |

Table 1: Mean dietary needs for macro nutrients and energy, per types of meal and individual ($qm_{nmi}$). B - Breakfast; L - Lunch; S - Snack; D - Dinner.

| | Child | | Elderly | | Others | |
|---|---|---|---|---|---|---|
| Nutrient | Day | Month | Day | Month | Day | Month |
| Carbohydrate (g) | 233.9 | 7017 | 217.4 | 6522 | 288.8 | 8664 |
| Protein (g) | 74.4 | 2232 | 78.8 | 2364 | 91.9 | 2757 |
| Fat (g) | 52 | 1560 | 55.1 | 1653 | 64.1 | 1923 |
| Energy (Kcal) | 1700 | 51000 | 1800 | 54000 | 2100 | 63000 |
| Calcium (mg) | 445 | 13350 | 1200 | 36000 | 1160 | 34800 |
| Iron (mg) | 7.1 | 212 | 8 | 240 | 11 | 330 |
| Magnesium (mg) | 78.8 | 2362.5 | 370 | 11100 | 344 | 10320 |
| Phosphorus (mg) | 333.8 | 10012.5 | 700 | 21000 | 920 | 27600 |
| Vitamine B12 ($\mu$g) | 0.8 | 22.5 | 2.4 | 72 | 2.3 | 68.4 |

Table 2: Mean dietary needs for nutrients and energy per type of individual, daily ($q'_{ni}$) and monthly ($q_{ni}$).

Computational results have not yet been reported. The commercial linear programming solver that we intend to use is Xpress-IVE [6]. MOS Files that contain the program implementation of the models and heuristics will be prepared to read data from CSV files which are subject to monthly or daily changes. To facilitate the use of our implementation by any layman, the process that allows one to change the data files will be performed using a small interface, developed specifically for this purpose in the Java language.

# 5 Conclusions and recommendations

Although no computational results are as yet available, we think that the linear programming approach presented in this work must successfully solve the distribution problems of donated products among the social solidarity institutions. This approach considers the dietary requirements of the individuals in need without favouring any institution in particular. The models proposed for the dry and fresh product distributions are similar, apart from the fact that the model for the fresh products considers the needs already met by the previous distributions over the month. If the computational experiment confirms our expectations, a donation of Xpress licenses to the Portuguese Federation of Food Banks will be extremely useful. In order to use the information about the dietary needs per type of individual in a more accurate way, we suggest that the form filled in by each institution should define when an individual is a child or an elderly person, and ask for the number of elderly receiving baskets.

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