Report on *Lotsizing and scheduling in BA Vidro*

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Problem presented by: *BA Vidro*

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1 Introduction

BA Vidro is a glass container industry that produces, develops and sells a wide range of glass containers for food, beverage, cosmetics and pharmaceutical companies. Currently it has five factories, equipped with eleven furnaces and thirty seven forming machines as illustrated in Figure 1.

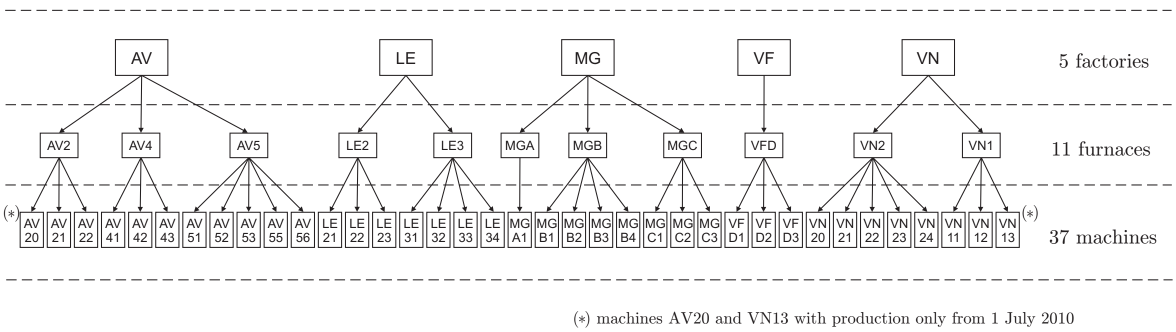

Figure 1: Production structure

The company produces around 1500 distinct products and deals with a very large volume of customer requests, and so it is essential to develop an efficient production planning.

Concerning the production management, the company has to deal with several characteristics and constraints inherent to its structure and equipment. The manufacturing process is continuous, *i.e.*, furnaces operate continuously seven days a week, except when they are being repaired. Color changeovers in furnaces and changeover of products in machines lead to a major cost in production. For instance, when there is a changeover in a machine there is a loss of the daily production of 45%, in average, in the corresponding machine. Because of the human labor power schedule, there cannot be changeovers of products on machines during the weekends and the holidays. On Fridays, at most one changeover of products is allowed in each factory, while two of them are allowed during other weekdays (except for specific large machines which can not be changed together with other machines). At a given moment, each machine can only produce one product and all the machines associated to the same furnace have to produce containers of the same color.

Several information is provided (input data), namely, product color and weight, monthly product sales forecast (12 months sales plan), initial and safety stocks, machines efficiency, furnaces capacity (measured in melted tonnes per day) and daily production rate of each article in each machine.

The production planning of *BA Vidro* is very complex since it combines production planning decisions (how much to produce of each product in each machine for each month) with scheduling decisions in order to satisfy to the maximum number of changeovers in each factory per day. Hence, it is a major challenge for the company to provide a significant improvement in production planning and scheduling.

The management of the stock level held by *BA Vidro* is crucial for the manufacture economics. Allowing the fluctuation in the level of stocks according to several changes in customers demand (seasonal variations in demand for glass packed products lead to corresponding variations in demand for many types of glass containers), it could maintain a better level of production. However, this company and other factories in general are constrained in the size of stocks that they can hold.

In summary, there are three main aspects to consider,

(i) maintain customer satisfaction, *i.e.*, satisfy customer demand and fulfill delivery deadlines;

(ii) optimize the capacity, maintaining production lines operating at its maximum level;

(iii) reduce the stocks.

The crucial question is: satisfying as much as possible conditions (i) and (ii), how can condition (iii) be improved? The company *BA Vidro* would like to find an efficient way to develop the production planning and scheduling, in order to reduce the costs resulting from storage.

Question presented by *BA Vidro*:

> What is the optimal stock level (measured in production days [1]) for the current company's Sales Plan? This problem leads to these three sub questions:
>
> - What is the optimal production batch for the current mix of the annual Sales Plan?

[1] Production days are (average stock of last 12 months/production of the last 12 months) $\times$ 365.

- How many job changes should the current sales plan require considering the existing Production Structure?
- What is the number of manufacturing changes appropriate to the relationship between "productive structure vs. the current mix of BA Sales Plan"?

Production planning problems have been intensively studied during the last decades. Several surveys on the subject with problem classifications have been proposed (*e.g.* [12, 13]). Integration of lotsizing with scheduling decisions has also been considered for decades (see [10]). The approach of solving production planning problems using mixed integer programming is more recent and a detailed and very complete survey can be found in [13].

In the glass industry, production planning and scheduling problems have also received a lot of attention in recent years. Due to its complexity, the system is decomposed into a two-level hierarchy planning structure: long-term and short-term levels. In [1] are explored extensions of lotsizing and scheduling problems that appear in both levels. The short-term production planning and scheduling problem coming from the glass container industry have been treated with several approaches, see *e.g.* [5, 8, 14, 15]. A variant of the variable neighborhood search (VNS) is introduced in [3] to tackle the production planning and scheduling problem that arises at the long-term planning level of the glass container industry. A single machine multi-product capacitated lotsizing with sequence-dependent startup times and costs is considered in [2]. A model for the single machine capacitated lotsizing and scheduling problem (CLSP) with sequence dependent startup times and startup costs is developed in [11], incorporating all the usual features of startup carryovers. In [4] a new set of constraints is added to the model to provide an exact formulation for this problem. The glass container industry production planning and scheduling problem is also studied in [6].

This report is organized as follows. In Section 2 an approach based on the decomposition of the problem into two related problem (the lotsizing and the scheduling problems) is proposed. The mathematical models are left to the Appendix. In Section 3 a summary of the results is given. Finally, we state some conclusions and recommendations in Section 4.

2 Lotsizing and Scheduling Approach

The problem presented by *BA Vidro* includes both medium-term and long-term decisions such as the number of production days of each product in each machine on each month, during one year, as well as short-term decisions such as the production scheduling taking into account the startups. Given the size of the problem (with more than one thousand products) and the long time horizon considered (one year), solving to optimality the lotsizing and the scheduling problems simultaneously seems an unreachable task. The classical approach considers separately these two problems: the lotsizing problem to establish the production plan and the scheduling problem to obtain the production sequence for each machine. The output of the lotsizing model is input of the scheduling problems (one for each factory and each month). The scheduling problem aims to find a production plan scheduling for a month in a given factory in order to satisfy startup constraints. If the startup constraints can not be satisfied, that information is passed to the lotsizing model which can be solved again considering a tighter bound on the maximum number of startups allowed in that factory.

In the next subsections a detailed description of each problem is presented. We introduce the following definitions:

- A machine is **setup** for an item i if the machine is prepared to produce that item.
- There is a **startup** in a given period when the machine is setup for an item in that period and was not setup for that item in the previous period.
- The **backlog** is the demand that is satisfied with delay.

2.1 Lotsizing problem

Given the physical productive structure of the company, the lotsizing problem considered consists in establishing the annual production plan in order to minimize the stock level, while satisfying the demands.

In the lotsizing problem there are several elements to consider, namely, the time horizon, inputs, constraints, decisions, that we enumerate next.

Time horizon

- Each time period is one month;
- The time horizon is one year.

Input

- Set of factories;
- Set of furnaces;
- Set of machines;
- Production rates for each product in each machine;
- Demands;
- Safety stocks;
- Initial stocks.

Constraints

- Satisfy the demand of each product in each month;
- Maintain the safety stock of each product in each month;
- Establish a correspondence between the color of each furnace and the color of the products to produce;
- Ensure that the production is not interrupted neither in machines nor in furnaces;
- Assure that the maximum number of startups per factory is not exceeded.

Decisions

- Number of production days of each color on each furnace in each month;
- Number of production days of each product on each machine in each month;
- Number of startups on each machine in each month.

The mathematical formulation of the corresponding problem is described in Appendix A.1.

Since it is assumed that the color changeover time in the furnaces do not depend on the sequence of the color and the startup of the machines do not depend on the sequence of products, a new model with less variables is proposed. In this new model the sequence of products (in the machine) and the sequence of colors (in the furnaces) is not considered, only the number of startups is counted. The mathematical formulation is described in Appendix A.2.

2.2 Scheduling on each factory

The input of the scheduling problem is given by the production plan obtained from a solution of the lotsizing problem. For each factory and for each month, the scheduling problem is to determine the sequence of colors and the sequence of tasks (or starting time of each task) inside the corresponding color in order to meet the production plan.

As in the previous problem, we present a list of elements to consider.

Time horizon

- The time unit is the day;
- The time horizon is one month.

Input

- Set of furnaces;
- Set of machines;
- Number of production days of each color on each furnace;
- Number of production days of each product on each machine;
- Initial color of each furnace.

Constraints

- Ensure that the number of allowed startups (that depends on the day of the week and the machine features) is not exceeded;

– Assure that each color/item processing period is completed;

– Allow only one startup for each one of the colors/products to be processed on each furnace/machine;

– Establish a correspondence between machine setups and product changeovers.

Decisions

– Sequence of colors for each furnace;

– Sequence of products for each machine.

The model used prevents solutions that do not satisfy the maximum number of startups per day by penalizing the additional startups. The mathematical formulation of the problem is presented in Appendix B.

2.3 Problem solving

In order to solve each problem the commercial software Xpress [9] was used. This software requires a mathematical model of the problem and the data input (see Figure 2).

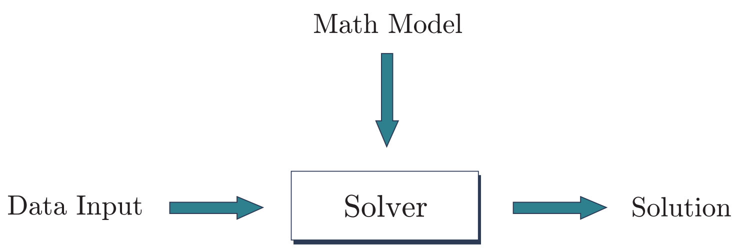

Figure 2: Solver

The mathematical models used can be found in the appendix.

First the lotsizing problem is solved. This model does not take into account the schedule of the production in each machine. Considering the production plan, the scheduling problem can be solved for each factory on each month. By using the scheduling model either a feasible schedule (a schedule satisfying the startup requirements) is found or information for the additional startups per day is obtained. In this last case that information can be used in the first model by restricting further the number of startups in the factories where the scheduling problem found no feasible solution (see Figure 3).

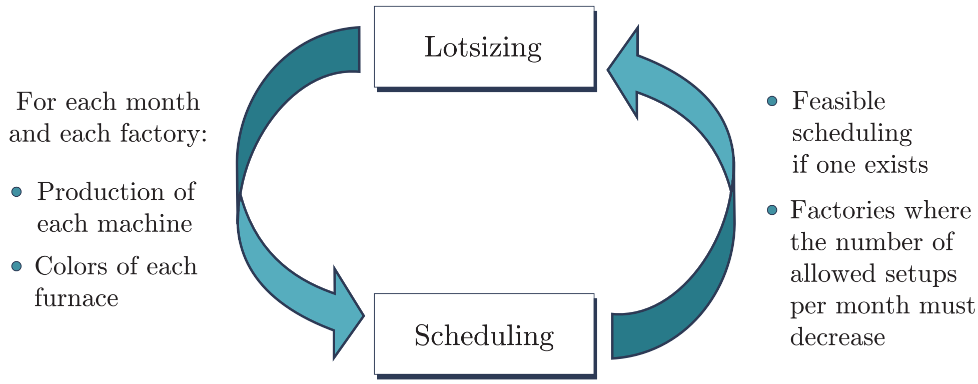

Figure 3: Lotsizing *versus* Scheduling

3 Results

Using the strategy presented in Section 2, the lotsizing problem has been solved. It was possible to show that the company has no capacity to satisfy all the forecasted demand, even ignoring some relevant constraints such as the furnaces capacity and the maximum number of setups per factory in each month and that there is no loss of production with machine startups and color changeovers. In that case more than 20000 tonnes of products were not possible to satisfy.

Next we present the results obtained when considering the scenario where we assume:

(i) the goal is to minimize the average stock level;

(ii) the number of setups per period at each factory should be lower than twice the number of working days in that period;

(iii) machines should not stop operating;

(iv) there is no loss on the furnace production capacity with color changeovers.

The results obtained using the model presented in Appendix A.2 for this scenario are summarize in Table 1.

Stock (measured in number of production days)	64,3
Initial stock (tonnes)	216.216
Final stock (tonnes)	155.416
Average monthly stock (tonnes)	139.730
Unsatisfied demand (tonnes)	107.728
Total production (tonnes)	793.216
Number of startups	1798

Table 1: Summary of results

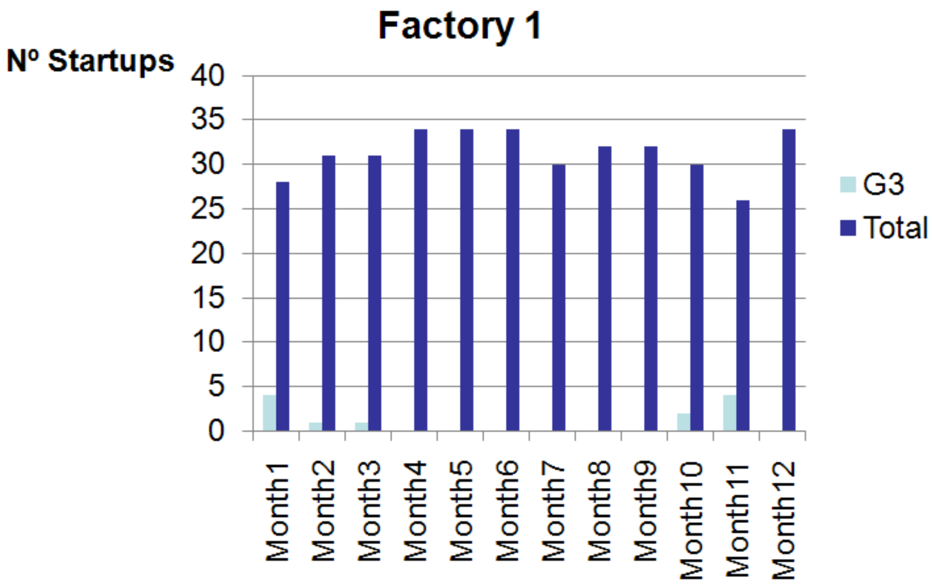

Figure 4: Number of startups in Factory 1

One can easily observe that the number of startups is very high (in order to minimize the stock level). In particular, the number of startups in Factory 4 is too high since this factory has only three machines. Hence there is a very high number of startups per machine on this factory. A future direction to obtain more interesting solutions could be to restrict the number of allowed startups on each factory accordingly to the number of machines. Also additional information on the color changeovers could be provided and the number of color changeovers could also be restricted.

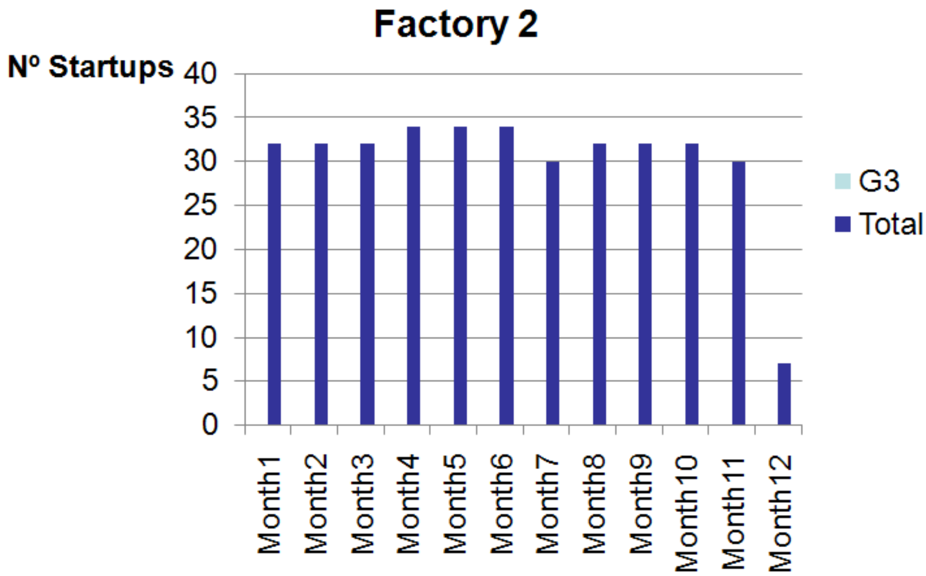

Figure 5: Number of startups in Factory 2

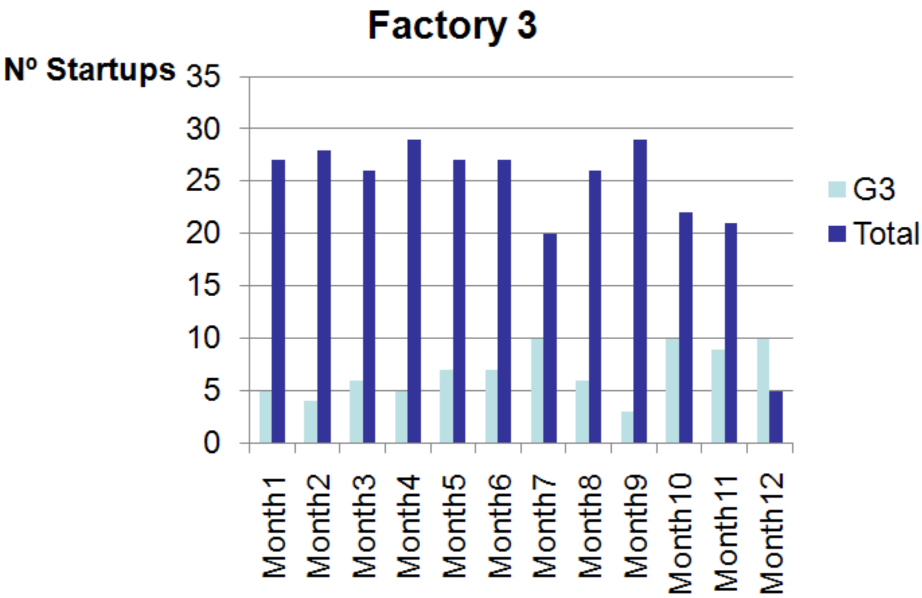

Figure 6: Number of startups in Factory 3

4 Conclusions and Recommendations

The problem presented by *BA Vidro* is a Lotsizing and Scheduling problem common to many companies in Portugal and around the world. This type of problems has been intensively studied

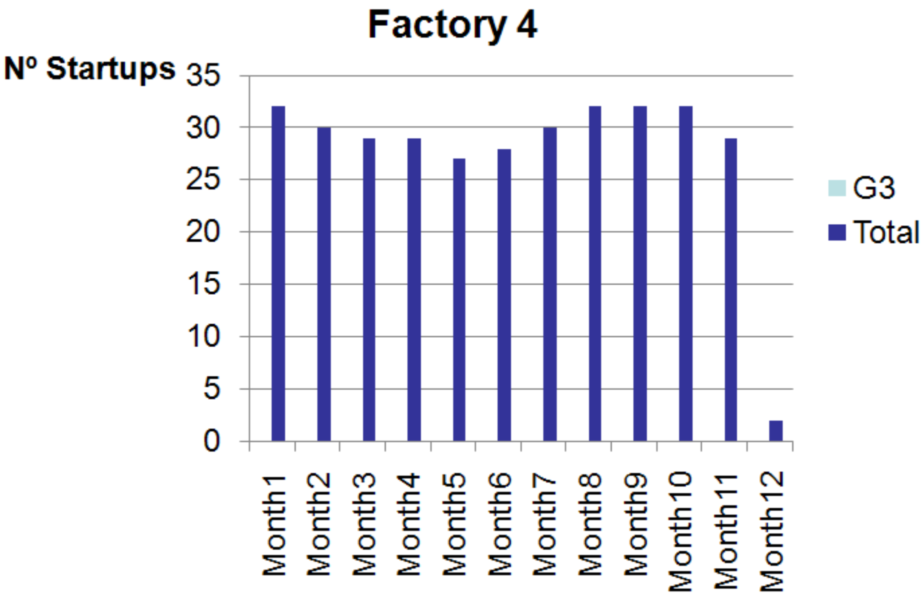

Figure 7: Number of startups in Factory 4

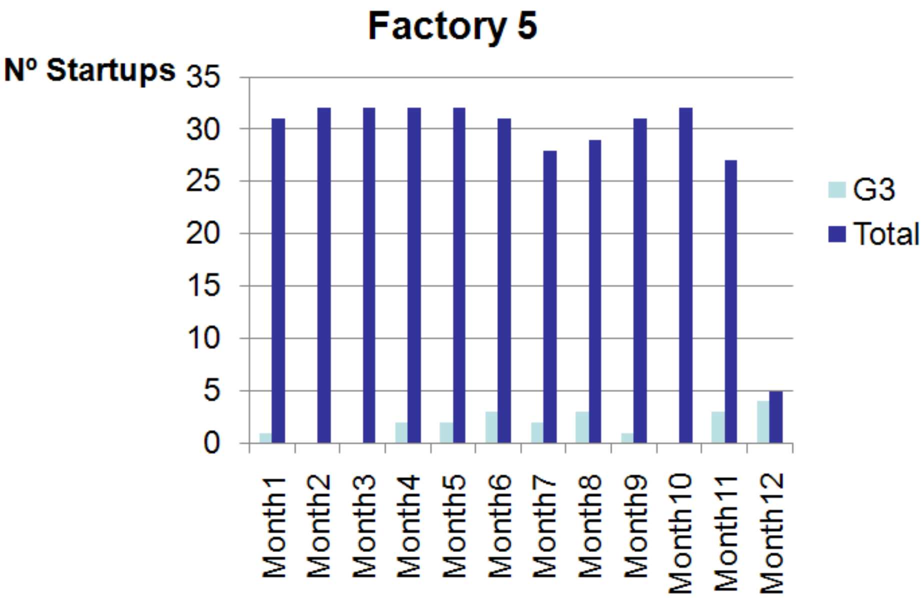

Figure 8: Number of startups in Factory 5

(by companies and research teams) in the past and, in particular, during the last decade. *BA Vidro* is a very interesting example of that by having a PhD thesis based on their production planning problems (see [1]). However, only recent specialized software has been able to provide an effective help to companies in their production planning activity. This help is usually limited since it is impossible to cover all real constraints in a model required by a software package. Also, the huge complexity of the problem restricts the set of possible approaches to solve the problem, for instance, by forcing to consider the two problems (lotsizing and scheduling) separately or to consider shortest time horizons.

Given the large economic relevance of the current production planning problems and the increase of their complexity by the expansion of the company during the last years, it would be advised that the company could maintain a permanent effort on improving their techniques to solve these production planning problems. Two directions for improvement can be easily stated. One is to improve the mathematical models provided in this report by progressively incorporating other main constraints of the company. Another direction is to consider intermediate planning horizons and to develop specialized software (or develop models) that integrates the two types of decisions. In both directions, from the models developed the company can have feedback on which constraints (technological and human workforce) have greater economic impact on the planning process. This modeling effort requires a deep commitment of the decision makers of the companies. Their expertise is crucial for the permanent improvement of the techniques used.

Appendix: Mathematical Formulations

A Lotsizing Problem

A.1 Lotsizing: changeovers

In this section we state the model formulation for the annual production plan described in Section 2.1, using the month as the time unit.

In order to formalize the production planning model we introduce the following notation.

Sets

P	set of products
L	set of colors
P_u	set of products with color u
G	set of factories
F	set of furnaces
K	set of machines
K^g	set of machines fed on factory g
K^f	set of machines fed by furnace f
K_3	set of large scale machines for which only one changeover is admissible per day
T	set of periods

Indices

i, j	product, $i, j \in P$
u, v	product color, $u, v \in L$
g	factory, $g \in G$
f	furnace, $f \in F$
k	machine, $k \in K$
t	period, $t \in T$

Parameters

p_i^k	producing rate of product i on machine k (tonnes per day)
η^k	efficiency of machine k
e_{uv}^f	wasted melted glass to setup furnace f from color u to color v (tonnes)
d_i^t	demand for product i at the end of period t (tonnes)
c^{ft}	capacity of furnace f in period t (tonnes)
$\underline{s}_i^t$	safety stock of product i at the end of period t (tonnes)
nd_t	number of days in period t
nwd_t	number of working days in period t (Monday through Friday)

Decision variables

x_i^{kt}	number of production days of product i on machine k in period t
γ_i^{kt}	production of product i on machine k in period t (tonnes)
s_i^t	stock of product i at the end of period t (tonnes)
r_i^t	backlog of product i at the end of period t (tonnes)
α_u^{ft}	$\begin{cases} 1 & \text{if the furnace } f \text{ is setup for color } u \text{ at the beginning of period } t \\ 0 & \text{otherwise} \end{cases}$
w_{uv}^{ft}	$\begin{cases} 1 & \text{if a startup change occurs on furnace f from color } u \text{ to color } v\,(\neq u) \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$
ℓ_u^{ft}	auxiliary variable to avoid subcircuits in the sequence of colors on furnace f in period t
z_u^{ft}	number of days that glass of color u is melted in furnace f in period t
y_i^{kt}	$\begin{cases} 1 & \text{if machine } k \text{ is setup for product } i \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$

Using the above notation, a mixed integer programming problem to establish the annual production plan in order to minimize the stock level, can be formulated as follows.

$$\text{Minimize} \quad \sum_{i \in P} \sum_{t \in T} s_i^t \tag{A.1}$$

$$\text{subject to} \quad s_i^t - r_i^t \;=\; s_i^{(t-1)} - r_i^{(t-1)} + \sum_{k \in K} \gamma_i^{kt} - d_i^t \qquad i \in P,\ t \in T \tag{A.2}$$

$$\sum_{k \in K^f} \sum_{i \in P} \left(\gamma_i^{kt} / \eta^k + 0.45 y_i^{kt} p_i^k \right) + \sum_{u \in L} \sum_{v \in L \setminus \{u\}} e_{uv}^f w_{uv}^{ft} \;\leq\; c^{ft} \qquad f \in F,\ t \in T \tag{A.3}$$

$$\gamma_i^{kt} \;\leq\; x_i^{kt} p_i^k \eta^k - 0.45 y_i^{kt} p_i^k \eta^k \qquad i \in P,\ t \in T,\ k \in K \tag{A.4}$$

$$\sum_{i \in P_u} x_i^{kt} \;=\; z_u^{ft} \qquad u \in L,\ f \in F,\ k \in K^f,\ t \in T \tag{A.5}$$

$$x_i^{kt} \;\leq\; n d_t y_i^{kt} \qquad i \in P,\ k \in K,\ t \in T \tag{A.6}$$

$$y_i^{kt} \;\leq\; \sum_{v \in L} w_{vu}^{ft} + \alpha_u^{ft} \qquad i \in P_u,\ f \in F,\ u \in L,\ k \in K^f,\ t \in T \tag{A.7}$$

$$\sum_{u \in L} \alpha_u^{ft} \;=\; 1 \qquad f \in F,\ t \in T \tag{A.8}$$

$$\alpha_u^{ft} + \sum_{v \in L} w_{vu}^{ft} \;=\; \alpha_u^{f(t+1)} + \sum_{v \in L} w_{uv}^{ft} \qquad u \in L,\ f \in F,\ t \in T \tag{A.9}$$

$$\ell_v^{ft} \;\geq\; \ell_u^{ft} + |P| w_{uv}^{ft} - (|P| - 1) - |P| \alpha_u^{ft} \qquad u \in L,\ v \in L \setminus \{u\},\ f \in F,\ t \in T \tag{A.10}$$

$$\underline{s}_i^t \;\leq\; s_i^t \qquad i \in P,\ t \in T \tag{A.11}$$

$$\sum_{u \in L} z_u^{ft} \;=\; n d_t \qquad f \in F,\ t \in T \tag{A.12}$$

$$\sum_{i \in P} \sum_{k \in K^g \setminus K_3} y_i^{kt} + 2 \sum_{i \in P} \sum_{k \in K^g \cap K_3} y_i^{kt} \;\leq\; 2nwd_t \qquad g \in G,\ t \in T \tag{A.13}$$

$$\gamma_i^{kt},\ s_i^t,\ r_i^t \geq 0 \tag{A.14}$$

$$\alpha_u^{ft},\ w_{uv}^{ft},\ y_i^{kt} \in \{0,1\} \tag{A.15}$$

$$x_i^{kt},\ \ell_u^{ft},\ z_u^{ft} \in \mathbb{Z}_+ \tag{A.16}$$

The objective function (A.1) corresponds to the minimization of the stock for all products throughout the production period.

Constraints (A.2) represent the inventory balances. At the end of each period t, the difference between the stock and the backlog for each product must be equal to the corresponding difference in the previous period plus the produced product minus the demand, for this period. Constraints (A.3) ensure that the total production on each furnace and in each period does not exceed the available melting capacity.

Constraints (A.4) bound the produced quantity of the product i on machine k in period t, considering the efficiency of machine k, the producing rate of product i on this machine and the loss of the daily production when there is a startup in a machine which is of 45%, in average.

Constraints (A.5) force machines fed by the same furnace to process each color for the same amount of time. Constraints (A.6) assures that if machine k produces product i in period t, then the number of production days, in this period, cannot exceed the duration of this period.

Constraints (A.7)-(A.10) establish the sequence of colors on each furnace in each period and keep track of the furnace configuration state by determining the color that a furnace is ready to process. Production of product i with color u can occur on machine k in period t if that machine is fed by a furnace f that is setup for u at the beginning of t or if at least one startup is performed for color u on this furnace. These conditions are assured by constraints (A.7). Constraints (A.8) assure that in the beginning of each period each furnace is setup for an initial color. Constraints (A.9) maintain and carry the setup configuration state of the furnace into the next period and balance the network flow. Constraints (A.10) avoid disconnected sub-tours.

Constraints (A.11) assure that the stock of product i at the end of period t must be greater or equal to the safety stock of this product in the corresponding period.

Constraints (A.12) ensure that each one of the furnaces is melted to some color during all the production days in each period.

The number of allowed startups depends on the day of the week. There cannot be product startups on machines during the weekends, on Fridays there could be only one startup and on the remaining days there may be two startups on different machines, except on specific large scale machines, which correspond to the set denoted by K_3. Constraints (A.13) bound the total of startups that occur during each period t, a month.

Finally, constraints (A.14)-(A.16) represent the integrality and nonnegativity constraints.

A.2 Lotsizing: startups

In this section a new model with less variables is proposed. In this model the sequence of products (in the machine) and the sequence of colors (in the furnaces) is not considered, only the number of startups is counted.

In order to count the number of startups it is necessary to know the first and the last produced product in each period (and the first and last color produced in each furnace). Hence, there is a startup at the beginning of a period if the product produced at the beginning of the period is different from the product produced at the end of the previous period.

The decision variables are listed below

Decision variables

x_i^{kt}	number of production days of product i on machine k in period t
γ_i^{kt}	production of product i on machine k in period t (tonnes)
s_i^t	stock of product i at the end of period t (tonnes)
r_i^t	backlog of product i at the end of period t (tonnes)
a_i^{kt}	$\begin{cases} 1 & \text{if machine } k \text{ is setup for product } i \text{ at the beginning of period } t \\ 0 & \text{otherwise} \end{cases}$
b_i^{kt}	$\begin{cases} 1 & \text{if machine } k \text{ is setup for article } i \text{ at the end of period } t \\ 0 & \text{otherwise} \end{cases}$
y_i^{kt}	$\begin{cases} 1 & \text{if machine } k \text{ is setup for article } i \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$
v_i^{kt}	$\begin{cases} 1 & \text{if machine } k \text{ is startup for article } i \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$
α_u^{ft}	$\begin{cases} 1 & \text{if furnace } f \text{ is setup for color } u \text{ at the beginning of period } t \\ 0 & \text{otherwise} \end{cases}$
β_u^{ft}	$\begin{cases} 1 & \text{if furnace } f \text{ is setup for color } u \text{ at the end of period } t \\ 0 & \text{otherwise} \end{cases}$
δ_u^{ft}	$\begin{cases} 1 & \text{if furnace } f \text{ is setup for color } u \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$
w_u^{ft}	$\begin{cases} 1 & \text{if furnace } f \text{ is startup for color } u \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$
z_u^{ft}	number of days that glass of color u is melted in furnace f in period t
y_i^{kt}	$\begin{cases} 1 & \text{if machine } k \text{ is setup for product } i \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$
e_u	loss of production when there is a changeover in a furnace for color u consider $e_u = 2$.

$$\text{Minimize} \quad \sum_{i\in P}\sum_{t\in T} s_i^t \tag{A.17}$$

$$\text{subject to} \quad s_i^t - r_i^t \;=\; s_i^{(t-1)} - r_i^{(t-1)} + \sum_{k\in K}\gamma_i^{kt} - d_i^t \qquad i\in P,\ t\in T \tag{A.18}$$

$$\sum_{k\in K^f}\sum_{i\in P}\left(\gamma_i^{kt}/\eta^k + 0.45 y_i^{kt}p_i^k\right) + \sum_{u\in L} e_u^f w_u^{ft} \;\le\; c^{ft} \qquad f\in F,\ t\in T \tag{A.19}$$

$$\gamma_i^{kt} \;\le\; x_i^{kt}p_i^k\eta^k - 0.45 v_i^{kt}p_i^k\eta^k \qquad i\in P,\ t\in T,\ k\in K \tag{A.20}$$

$$\sum_{i\in P_u} x_i^{kt} \;=\; z_u^{ft} \qquad u\in L,\ f\in F,\ k\in K^f,\ t\in T \tag{A.21}$$

$$x_i^{kt} \;\le\; nd_t y_i^{kt} \qquad i\in P,\ k\in K,\ t\in T \tag{A.22}$$

$$v_i^{k1} = y_i^{k1} \qquad i\in P,\ k\in K \tag{A.23}$$

$$v_i^{kt} \le y_i^{kt} \qquad i\in P,\ k\in K,\ t\in T,\ t>1 \tag{A.24}$$

$$v_i^{kt} \ge y_i^{kt} - a_i^{kt} \qquad i\in P,\ k\in K,\ t\in T \tag{A.25}$$

$$v_i^{kt} \ge y_i^{kt} + a_i^{kt} - b_i^{k,t-1} - 1 \qquad i\in P,\ k\in K,\ t\in T,\ t>1 \tag{A.26}$$

$$a_i^{kt} \le y_i^{kt} \qquad i\in P,\ k\in K,\ t\in T \tag{A.27}$$

$$b_i^{kt} \le y_i^{kt} \qquad i\in P,\ k\in K,\ t\in T \tag{A.28}$$

$$a_i^{kt} + b_i^{kt} \le 2y_i^{kt} - \frac{1}{nd_t}\sum_{j,j\neq i} y_j^{kt} \qquad i\in P,\ k\in K,\ t\in T \tag{A.29}$$

$$\sum_{i\in P} a_i^{kt} = 1 \qquad k\in K,\ t\in T \tag{A.30}$$

$$\sum_{i\in P} b_i^{kt} = 1 \qquad k\in K,\ t\in T \tag{A.31}$$

$$w_u^{f1} = \delta_u^{f1} \qquad u\in L, f\in F \tag{A.32}$$

$$w_u^{ft} \leq \delta_u^{ft} \qquad u \in L, f \in F,\ t \in T,\ t > 1 \tag{A.33}$$

$$w_u^{ft} \geq \delta_u^{ft} - \alpha_u^{ft} \qquad u \in L, f \in F,\ t \in T \tag{A.34}$$

$$w_u^{ft} \geq \delta_u^{ft} + \alpha_u^{ft} - \beta_u^{f,t-1} - 1 \qquad u \in L, f \in F,\ t \in T,\ t > 1 \tag{A.35}$$

$$\alpha_u^{ft} \leq \delta_u^{ft} \qquad u \in L, f \in F,\ t \in T \tag{A.36}$$

$$\beta_u^{ft} \leq \delta_u^{ft} \qquad u \in L, f \in F,\ t \in T \tag{A.37}$$

$$\alpha_u^{ft} + \beta_u^{ft} \leq 2\delta_u^{ft} - \frac{1}{nd_t} \sum_{l, l \neq u} \delta_l^{ft} \qquad u \in L, f \in F,\ t \in T \tag{A.38}$$

$$\sum_u \alpha_u^{ft} = 1 \qquad f \in F,\ t \in T \tag{A.39}$$

$$\sum_u \beta_u^{ft} = 1 \qquad f \in F,\ t \in T \tag{A.40}$$

$$a_i^{kt} \leq \alpha_u^{ft} \qquad u \in L, i \in P_u, f \in F, k \in K^f, t \in T \tag{A.41}$$

$$b_i^{kt} \leq \beta_u^{ft} \qquad u \in L, i \in P_u, f \in F, k \in K^f, t \in T \tag{A.42}$$

$$y_i^{kt} \leq \delta_u^{ft} \qquad u \in L, f \in F, i \in P_u, , k \in K^f, t \in T \tag{A.43}$$

$$\underline{s}_i^t \ \leq \ s_i^t \qquad i \in P,\ t \in T \tag{A.44}$$

$$\sum_{u \in L} z_u^{ft} \ = \ nd_t - \sum_{u \in L} e_u w_u^{ft} \qquad f \in F,\ t \in T \tag{A.45}$$

$$\sum_{i \in P} \sum_{k \in K^g \setminus K_3} v_i^{kt} + 2 \sum_{i \in P} \sum_{k \in K^g \cap K_3} v_i^{kt} \ \leq \ 2nwd_t \qquad g \in G,\ t \in T \tag{A.46}$$

$$\gamma_i^{kt},\ s_i^t,\ r_i^t \geq 0 \tag{A.47}$$

$$a_i^{kt},\ b_i^{kt},\ v_i^{kt},\ \alpha_u^{ft},\ \alpha_u^{ft},\ w_u^{ft},\ \delta_u^{ft},\ y_i^{kt} \in \{0, 1\} \tag{A.48}$$

$$x_i^{kt},\ z_u^{ft} \in \mathbb{Z}_+ \tag{A.49}$$

The objective function (A.17) and constraints (A.18), (A.19), (A.21), (A.44) and (A.46) are analogous to (A.1), (A.2), (A.5), (A.11) and (A.13), respectively.

In constraints (A.20) it is given the produced quantity of the product i on machine k in period t, considering the efficiency of machine k, the producing rate of product i on this machine and the loss of the daily production when there is a changeover in a machine which is of 45%, in average.

Constraints (A.22) assures that if machine k produces product i in period t, then the number of production days, in this period, cannot exceed the duration of this period.

Constraints (A.23)-(A.31) count the number of startups for each product on each machine, in each period, and keep track of the machine configuration state by determining the product that a machine is ready to process. Constraints (A.23) ensure that, in the first period, all the setups correspond to startups. Constraints (A.24) ensure that if, in a period t, there is a startup for product i in machine k, then there is a setup for this product in this machine. Constraints (A.25) assure that, during period t, if there exist a setup for a product i different from the initial item, then there exist a startup for item i during this period. Constraints (A.26) guarantee that if the setup for the initial item, i, in a period t is different from the final item setup in the previous period, then there must be a startup for product i in this period. Constraints (A.27) ensure that if machine k is setup for product i at the beginning of period t, then it is setup for product i in period t. Constraints (A.28) assure that if machine k is setup for product i at the end of period t, then it is setup for product i in period t. Constraints (A.29) ensure that if, in a period t, machine k produces two different products, then the initial product setup and the final product setup must be different. Constraints (A.30) and (A.31) assure that in the beginning and in the end of each period each machine is setup for an item. Constraints (A.43) ensure that there can be a setup for an item only if the furnace is setup for the color of that item. Constraints (A.32)-(A.40) are identical to (A.23)-(A.31) where the roles of products/machine and colors/furnace are interchanged.

Constraints (A.41) assure that can occur a setup on machine k for product i with color u in the beginning of period t if that machine is fed by a furnace f that is setup for u at the beginning of t. Constraints (A.42) is identical but for the end of the period.

Constraints (A.45) ensure that each one of the furnaces is melted to some color during all the production days in each period minus the number of lost days by color changeovers.

Finally, constraints (A.47)-(A.49) represent the integrality and nonnegativity constraints for the variables.

B Scheduling for One Factory

In this section we state the model formulation for the monthly production plan described in Section 2.2. It is a short-term scheduling problem to provide the processing sequence of products on machines and the sequence of the colors on furnaces for a single factory, using the day as the time unit. Although we analyzed the production plan for a period of one month, different periods could be considered.

In order to formalize the short-term production planning model, for a given factory, we introduce the following notation.

Sets

P	set of products
L	set of colors
P_u	set of products with color u
F	set of furnaces
K	set of machines
K^f	set of machines fed by furnace f
K_3	set of large scale machines for which only one changeover is admissible per day
N	set of production days, $N = \{1, 2, \ldots, nd\}$, where $\vert N\vert = nd$
WE	subset of N corresponding to weekend days (Saturdays and Sundays)
FR	subset of N corresponding to Fridays

Input

n_u^f number of production days of color u on furnace f

n_i^k number of production days of product i on machine k

min^f smallest color production period on furnace f, $min^f = \min_{u \in L}\{n_u^f : n_u^f > 0\}$

min^k smallest product processing period on machine k, $min^k = \min_{i \in P}\{n_i^k : n_i^k > 0\}$

$$\alpha_u^f \begin{cases} 1 & \text{if } u \text{ is the initial color of furnace } f \\ 0 & \text{otherwise} \end{cases}$$

Indices

i, j product, $i, j \in P$

u, v product color, $u, v \in L$

f furnace, $f \in F$

k machine, $k \in K$

n period, $n \in N$

Decision variables

$$x_{ij}^{kn} \begin{cases} 1 & \text{if occurs a startup on machine } k \text{ from product } i \text{ to product } j \text{ at instant } n \\ 0 & \text{otherwise} \end{cases}$$

$$y_{uv}^{fn} \begin{cases} 1 & \text{if occurs a startup on furnace } f \text{ from color } u \text{ to color } v \text{ at instant } n \\ 0 & \text{otherwise} \end{cases}$$

$$\delta^{kn} \begin{cases} 1 & \text{if occurs a startup on machine } k \text{ at instant } n \\ 0 & \text{otherwise} \end{cases}$$

π^n auxiliary variables that penalize additional startups at each instant n

Using the above notation, a mixed integer programming problem to provide a feasible scheduling solution, if one exists, can be formulated as follows.

$$\text{Minimize} \quad \sum_{n\in N} \pi^n \tag{B.1}$$

$$\text{subject to} \quad \sum_{v,v\neq u} y_{uv}^{fn_u^f} = 1 \qquad f\in F,\ u\in L,\ \alpha_u^f = 1,\ n_u^f < nd \tag{B.2}$$

$$\sum_{u,u\neq v} y_{uv}^{fn} = \sum_{\substack{u,u\neq v \\ n+n_v^f+n_u^f \leq nd}} y_{vu}^{f(n+n_v^f)} \qquad f\in F,\ v\in L,\ n+n_v^f \leq nd - min^f \tag{B.3}$$

$$\sum_{u} \sum_{v,v\neq u} y_{uv}^{f(nd-n_v^f)} = 1 \qquad f\in F,\ min^f < nd \tag{B.4}$$

$$\sum_{n} \sum_{u,u\neq v} y_{uv}^{fn} + \alpha_v^f = 1 \qquad f\in F,\ v\in L,\ n_v^f > 0 \tag{B.5}$$

$$\sum_{i} \sum_{j,j\neq i} x_{ij}^{kn_i^k} = 1 \qquad k\in K,\ min^k < nd \tag{B.6}$$

$$\sum_{i,i\neq j} x_{ij}^{kn} = \sum_{\substack{i,i\neq j \\ n+n_j^k+n_i^k \leq nd}} x_{ji}^{k(n+n_j^k)} \qquad k\in K,\ j\in P,\ n+n_j^k \leq nd - min^k \tag{B.7}$$

$$\sum_{i} \sum_{j,j\neq i} x_{ij}^{k(nd-n_j^k)} = 1 \qquad k\in K,\ min^k < nd \tag{B.8}$$

$$\sum_{n} \sum_{i,i\neq j} x_{ij}^{kn} + \sum_{i,i\neq j} x_{ji}^{kn_j^k} = 1 \qquad k\in K,\ j\in P,\ n_j^k > 0 \tag{B.9}$$

$$x_{ij}^{kn} \leq \sum_{v} y_{uv}^{fn'} \qquad \begin{array}{l} j\in P,\ u\in L,\ i\in P_u,\ k\in K^f, f\in F \\ n,\ n'\in N : \{n\leq n',\ n-n_i^k \leq n'-n_u^f\} \end{array} \tag{B.10}$$

$$\delta^{kn} \geq \sum_{i\in P} \sum_{j\in P\setminus\{i\}} x_{ij}^{kn} \qquad k\in K,\ n\in N \tag{B.11}$$

$$\sum_{k\in K} \delta^{kn} \leq \pi^n \qquad n\in WE \tag{B.12}$$

$$\sum_{k\in K} \delta^{kn} \leq 1+\pi^n \qquad n\in FR \tag{B.13}$$

$$\sum_{k \in K \setminus K_3} \delta^{kn} + 2 \sum_{k \in K_3} \delta^{kn} \ \leq \ 2 + \pi^n \qquad n \in N \setminus (WE \cup FR) \tag{B.14}$$

$$x_{ij}^{kn}, \ y_{uv}^{fn}, \ \delta^{kn} \in \{0, 1\}, \ \pi^n \geq 0 \tag{B.15}$$

The sequence of colors on each furnace is established by (B.2)-(B.5). Constraints (B.2) assure that in the beginning of the scheduling period each furnace is setup for the initial color, u, and it will occur a startup to another color after the corresponding processing period. Constraints (B.3) are related to the flow conservation constraints and ensure that if a furnace starts to produce glass color v at instant n, after the respective processing period the furnace will be startup to another color. This color changeover assumes that the remaining time period allows the new color to be full processed. Constraints (B.4) assure that each furnace finishes the production period (a month) with a specific color. Constraints (B.5) force to exist only one startup for each one of the colors to be processed on each furnace.

The sequence of tasks on each machine is established by (B.6)-(B.9). Constraints (B.6) assure that in the beginning of the scheduling period each machine is setup to produce a specific product and it will occur a startup to another product after the corresponding production period. Constraints (B.7) are related to the flow conservation constraints and ensure that if a machine starts to produce product j at instant n, after the respective processing period the machine will startup to another product. This product changeover assumes that the remaining time period allows the new product to be full processed. Constraints (B.8) assure that each machine finishes the production period (a month) with a specific product. Constraints (B.9) force to exist only one startup for each one of the products to be processed on each machine.

Constraints (B.10) guarantee that during a production period of a given product there cannot exist color changeovers.

Constraints (B.11) relate machine startups with product changeovers, assuring that if occurs a product changeover on a machine k at an instant n, then it occurs a startup in this machine at the same instant.

Variables π^n are auxiliary variables that penalize additional startups at each instant n. The number of allowed startups depend on the day of the week. There cannot be startups on machines during the weekends, on Fridays there could be only one startup and on the remaining days there may be two startups on different machines, except on specific large scale machines, which correspond to the set denoted by K_3. Constraints (B.12), (B.13) and (B.14) correspond, respectively, to weekend, Fridays and to the remaining days.

Constraints (B.15) represent the integrality and nonnegativity constraints.

The purpose of the presented model is to obtain a short-term scheduling without additional startups. The objective function (B.1) minimizes the sum of additional startups throughout the production period, *i.e.*, the sum of the variables π^n. If the optimal value of this problem is different of zero, we conclude that, for the input data, a feasible schedule does not exist.

Remark: If the original colors on each furnace, α_u^f, are not given as input parameters, then constraints (B.2) and (B.5) can be written, respectively, as

$$\sum_u \sum_{v, v \neq u} y_{uv}^{fn_u^f} = 1 \qquad f \in F \tag{B.16}$$

$$\sum_n \sum_{u, u \neq v} y_{uv}^{fn} + \sum_{u, u \neq v} y_{vu}^{fn_v^f} = 1 \qquad f \in F,\ v \in L,\ n_v^f > 0. \tag{B.17}$$

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